Exercise 34

For the following exercises, find the average rate of change of each function on the interval specified.

$$k(t) = 6t^2 + \frac{4}{t^3} \text{ on } [-1, 3]$$

Solution

The average rate of change of the function between $t = -1$ and $t = 3$ is

$$\begin{aligned}
\frac{k(3) - k(-1)}{3 - (-1)} &= \frac{\left[6(3)^2 + \frac{4}{(3)^3}\right] - \left[6(-1)^2 + \frac{4}{(-1)^3}\right]}{3 + 1} \\
&= \frac{\left[6(9) + \frac{4}{(27)}\right] - \left[6(1) + \frac{4}{(-1)}\right]}{4} \\
&= \frac{\left(54 + \frac{4}{27}\right) - (6 - 4)}{4} \\
&= \frac{54 + \frac{4}{27} - 2}{4} \\
&= \frac{52 + \frac{4}{27}}{4} \\
&= \frac{52}{4} + \frac{4}{4(27)} \\
&= 13 + \frac{1}{27} \\
&= \frac{13(27) + 1}{27} \\
&= \frac{352}{27}.
\end{aligned}$$